National Defence Défense nationale

ASSISTANT DEPUTY MINISTER (DEFENCE RESEARCH AND DEVELOPMENT CANADA)

CANADIAN ARMED FORCES
FORCES ARMÉES CANADIENNES

DRDC RDDC

Statistical Failure Analysis of C3 Howitzer Barrels

Stephen Weber (CAN)

18 October 2022

NATO OR&A Conference

Canada

Overview

Canadian Army maintains a fleet of **~100** C3 Howitzers
Goal: Determine the mean lifetime of a cannon tubes
Challenge: Only **10%** of the tubes have failed

- *Engineering studies* employing X-ray diffraction and finite element analyis examine the root cause of the crack formation
    - **Cannot answer** broader questions about failure rates

- The *small sample size* of failures motivates the application of *Bayesian* simulation techniques over frequentist calculations

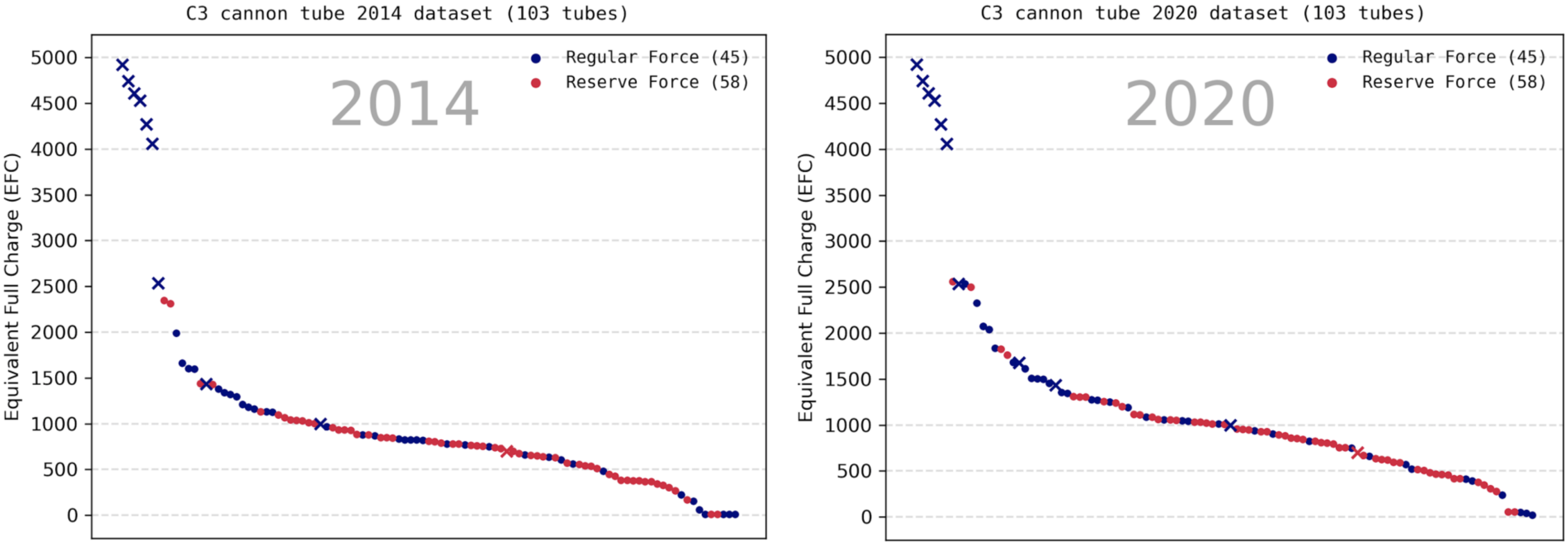
<u>Purpose of this study</u>:

- Determine probabilistic answers to questions surounding cannon tube lifetimes
- Provide fleet management guidance

PART 1
Qualitative analysis of the C3 howitzer fleet

Dataset visualization

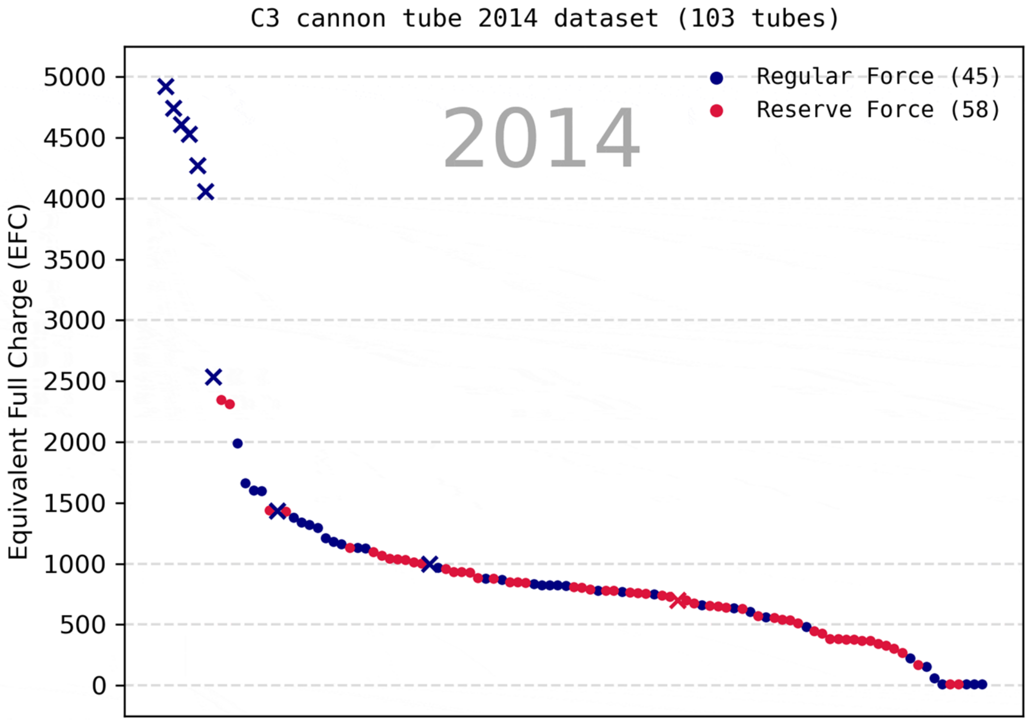
Each **point / cross** represents a **uncracked / cracked** tube



- Two inspections were made six years appart, in 2014 and in 2020
    - Obtain a rudimentary measure of the **usage rate** of the fleet
- Overall, the **total usage** of the majority of tubes *has not significantly increased* between 2014 and 2020

Dataset animation

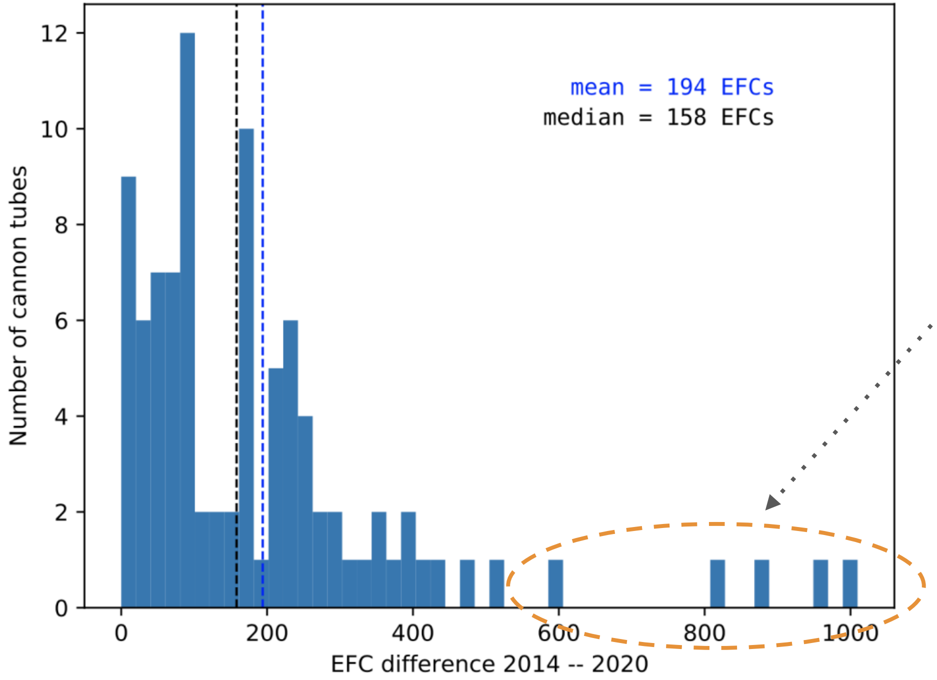
Each **point / cross** represents a **uncracked / cracked** tube



- This animation shows the *rate* at which the cannon tube usage (EFC) increased over the **6 year period** from **2014 - 2020**

Fleet usage rate

- Tube **usage / year** varies across the fleet, as is shown by the distribution of **EFC difference**
  - 2014 median = 780 EFC
  - 2020 median = 945 EFC

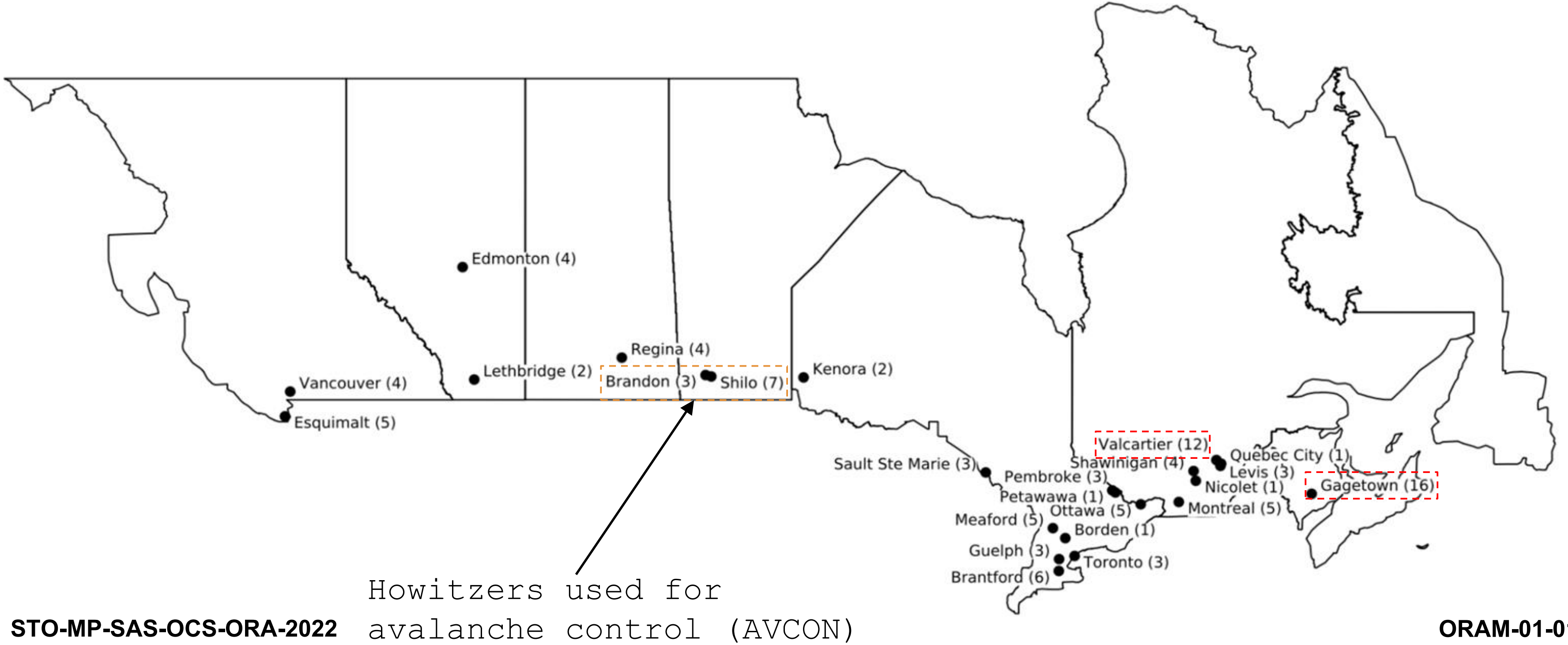


Majority of the tubes saw *relatively low usage* over the 6 year period:

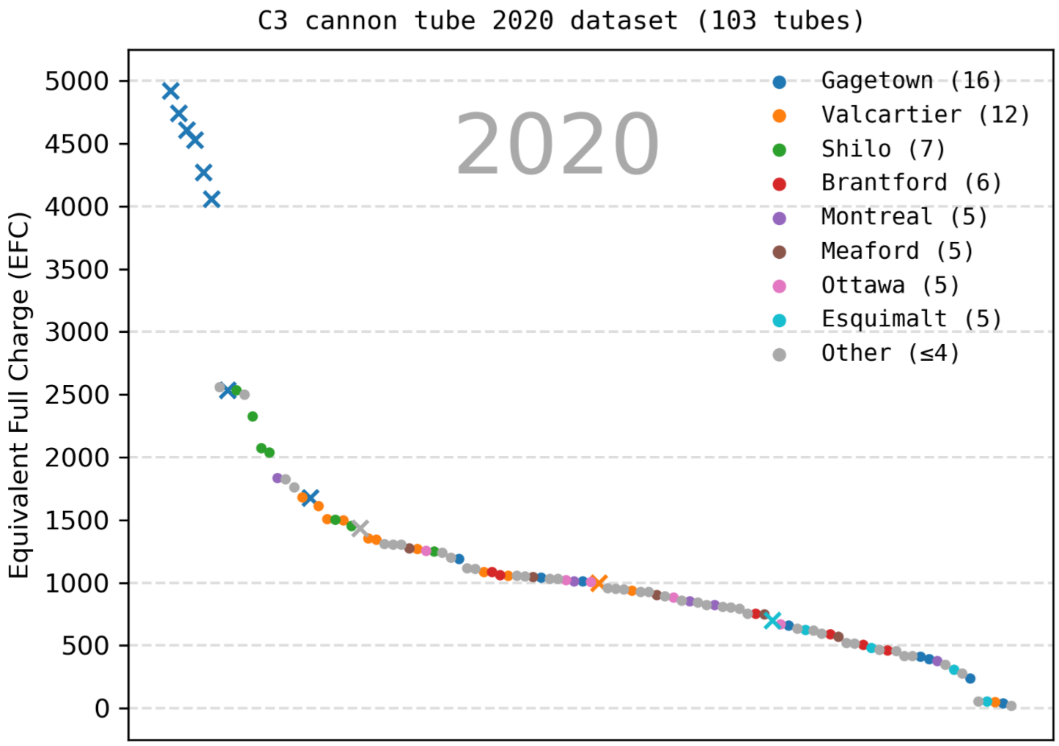
- **6** tubes have >**500** EFCs in the 6 years
  - These 6 tubes account for **26%** of the total usage
- **20** tubes have <**50** EFCs in the 6 years

Fleet location map

- The 2020 dataset includes **location** and **unit** information for each of the 103 tubes
- This map shows where the cannon tubes are *located across Canada*
  - The **number in parentheses** is the number of tubes at a given location



Fleet location information



- **8** of the **11** cracked tubes are in **Gagetown**
  - These are also the 8 *highest EFC* cracked tubes
- There is only one example of a cracked tube with ***less EFC*** than the *mean population EFC*
- The **Shilo** (AVCON) tubes have *above average usage* but **none are cracked**

Fleet location information

1RCHA fire the C3 Howitzer at a target in the mountains as part of avalanche control in Rogers Pass, BC.

Photo SLt Michael Dery

SHILO STAG NEWSPAPER Volume 57 Issue 24

- 10 tubes have been identified as having been recently used for avalanche control
- These 10 tubes (7 Shilo + 3 Brandon) *all have high usage of 1900 EFCs on average and there are no cracks*
- These **AVCON** tubes are important to monitor going forward in order to better understand the failure process
  - Currently there are *no uncracked tubes* with usage in the range of **2500 - 4000 EFCs**

PART 2
Statistical analysis of the crack formation process

Analysis method - Weibull Distribution

The crack failure process is modeled with the ***Weibull distribution***

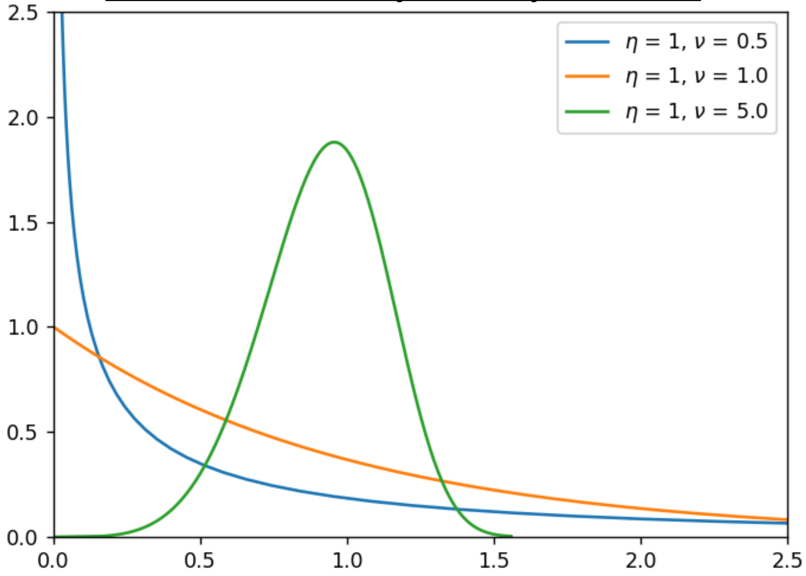
$$f(t;\nu,\eta) = \begin{cases} \frac{\nu}{\eta}\left(\frac{t}{\eta}\right)^{\nu-1} e^{-(t/\eta)^{\nu}} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

t is time, in our case this is **EFC**
ν is the shape parameter

- ***ν* < 1**, failure rate decreases over time
  - *Infant mortality process*
- ***ν* = 1**, exponential decay, constant failure rate
- ***ν* > 1**, failure rate increases over time
  - *Wearout process*

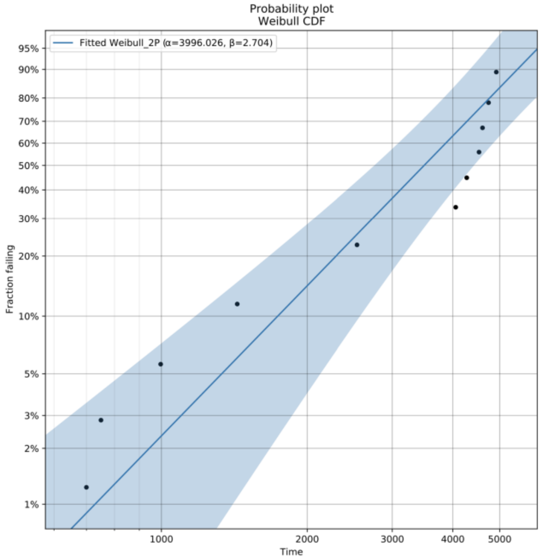
η is the scale parameter, this will be O(1000) in our case since **EFC** of the tubes is typically O(1000)

Weibull Probability Density Function



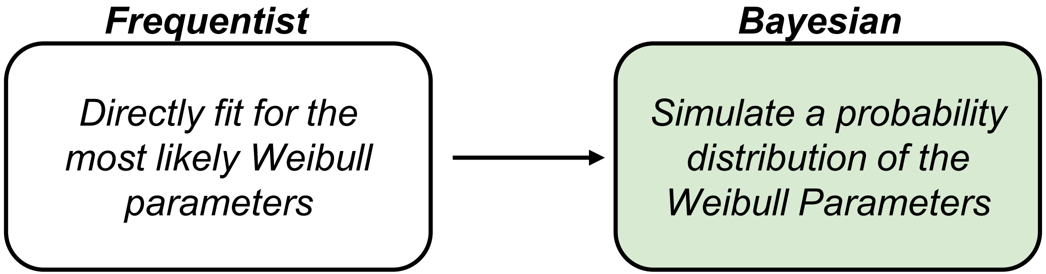
Analysis method - Frequentist approach

- Our data is mostly what is called *right censored*
- This means that **we don't know** when the majority of the tubes will fail because they haven't failed yet
- To analize the available data, the *freqentist approach* would be to directly fit the Weibull distribtuion, as shown bellow



ν = 2.7 (Shape)
η ~ 4000 (Scale)

- With heavily right censored data a ***Bayesian approach*** will provide more useful results



Defining the likelihood function

The **CDF** of the Weibull dist. is the *probability* that a tube will have cracked after **t** accumulated EFCs

$$F(t;\nu,\eta) = \begin{cases} 1 - e^{-(t/\eta)^{\nu}} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

The probability that the tube isn't cracked after time **t** is then **1 - CDF(t)**, this is referred to as the survival function **S**

With these two functions we can write down a *likelihood function* that depends on the Weibull parameters and on the cannon tube data

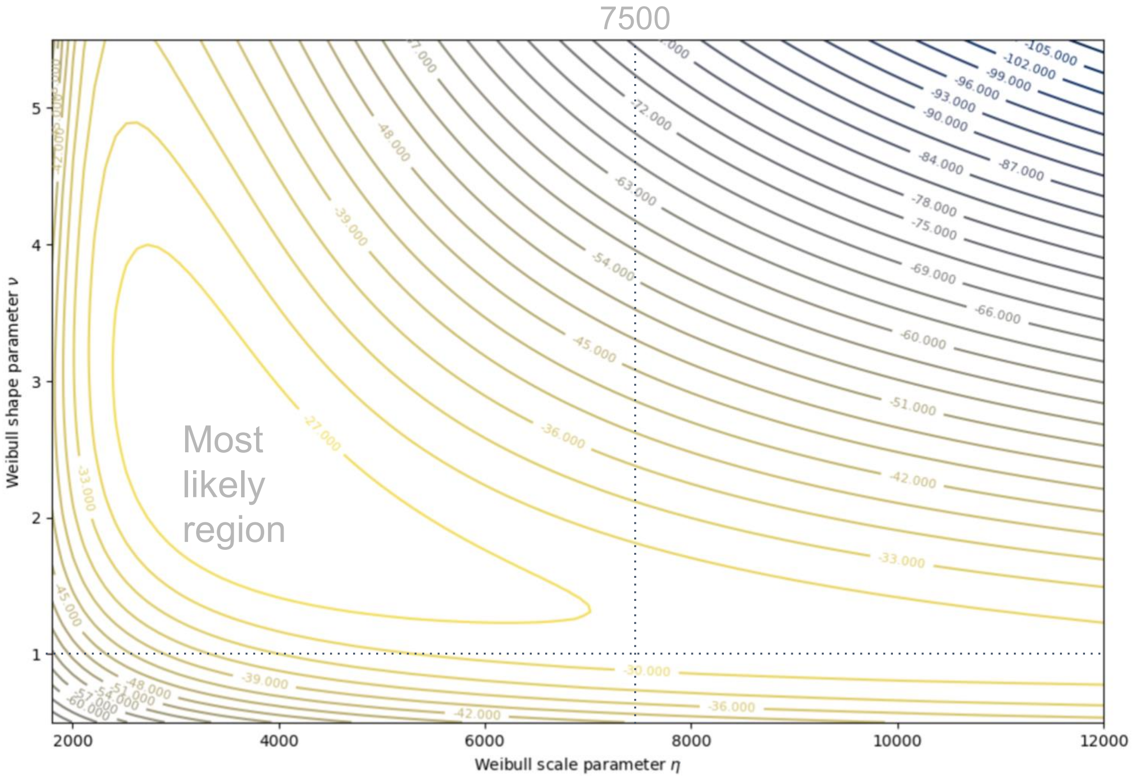
$$L(\{t_i\}|\nu,\eta) = \prod_{i \in \text{cracked}} F(t_i;\nu,\eta) \prod_{i \in \text{uncracked}} S(t_i;\nu,\eta)$$

t (time/EFC)
v (shape)
η (scale)

Failure function (CDF)

Survival function (1 - CDF)

Likelihood Function



$$L(\{t_i\}|\nu,\eta) = \prod_{i \in \text{cracked}} F(t_i;\nu,\eta) \prod_{i \in \text{uncracked}} S(t_i;\nu,\eta)$$

- These contours show the **value of the likelihood function** in the Weibull parameter space
- Observe that there is a **large central contour** where the likelihood function has a similar value
- This is evidence that a *single point maximum likelihood* approach **will not be very representative** of the complete picture

t (time/EFC)
v (shape)
η (scale)

Prior probability distribution

- The **prior probability distribution** represents our initial modeling assumptions
- Want a *weakly informative prior*
  - Limit bias and let the **data "speak for itself"**
- The prior should assign **equal probabilities** to the cases where the *Weibull shape parameter* ($\boldsymbol{\nu}$) is less than or greater than 1
- Similarly it should assign **equal probabilities** to cases where the **MTTCF** is less than or greater than 7500 EFCs
- The functional form of the prior is chosen to be a **Gamma distribution**
  - This is chosen because the Gamma distribution is the so-called *conjugate prior* of the exponential distribution, which appears in our **likelihood function**
  - The *conjugate prior* is analogous to an **eigenfunction** of the Bayes' theorem for a particular choice of likelihood function

Bayes' theorem

$$p(\nu, \eta | \{t_i\}) \propto L(\{t_i\} | \nu, \eta)\, p(\nu, \eta),$$

Gamma distribution

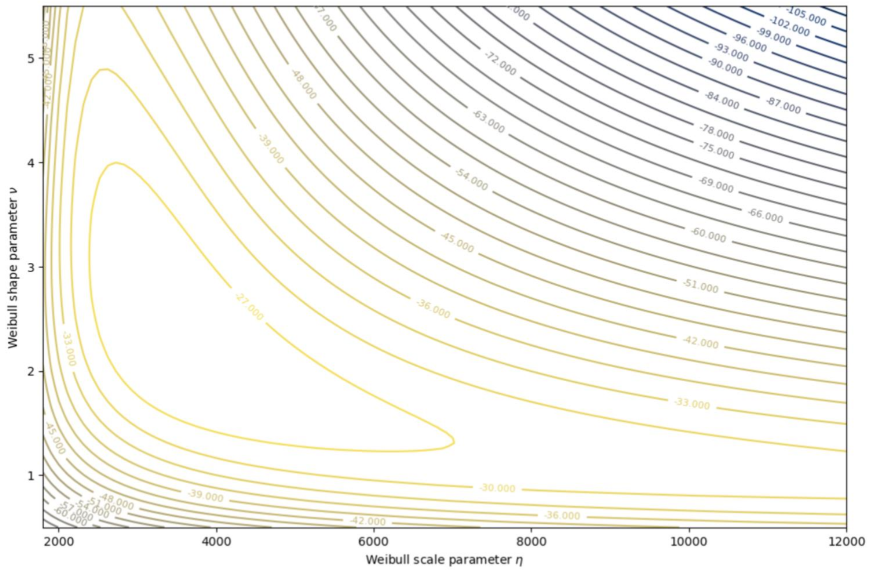
$$f(x; a, b) = \frac{1}{\Gamma(a)\, b^a} x^{a-1} e^{\frac{x}{b}},$$

Prior

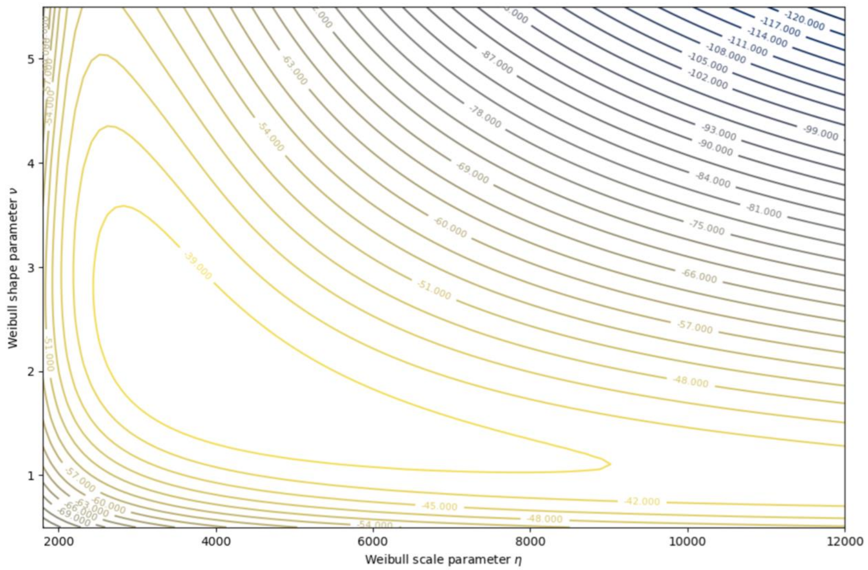
$$p(\nu, \bar{t}; a, b, c, d) = \left(\frac{1}{\Gamma(a)\, b^a} \nu^{a-1} e^{\frac{\nu}{b}}\right)\left(\frac{1}{\Gamma(c)\, d^c} \bar{t}^{c-1} e^{\frac{\bar{t}}{d}}\right).$$

Prior probability distribution

Likelihood only



Likelihood + prior



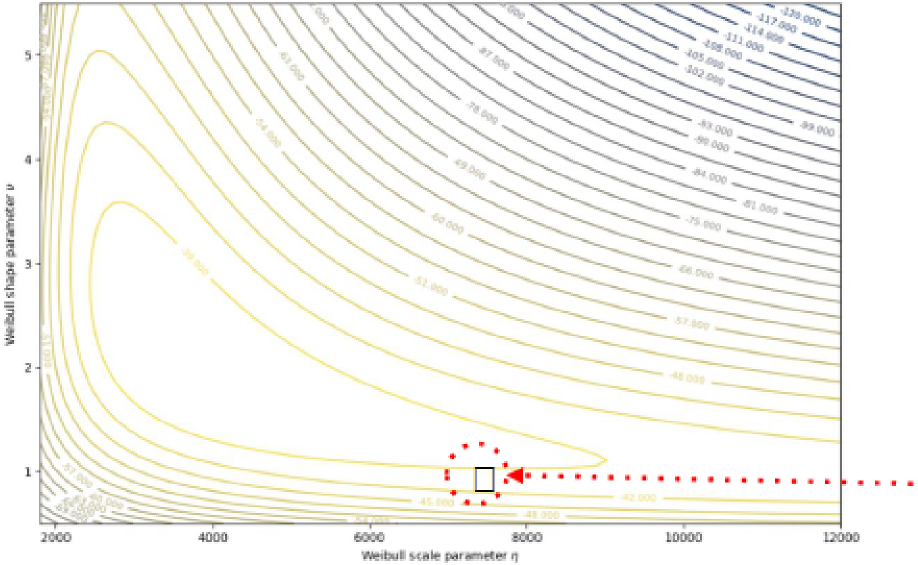
Including the prior:

- Higher scale parameter values (η ~ 7500 EFCs) are more likely when including the prior.
- The central contour has also moved somewhat down towards shape parameter $\nu = 1$.
- The feature that a *large area of parameter space has a similar likelihood* is even more true when including the prior, further motivating the Bayesian approach

Metropolis-Hastings Algorithm

Perform a *Markov Chain Monte Carlo* (MCMC) simulation using the likelihood function and a process called the **Metropolis-Hastings (MH)** algorithm

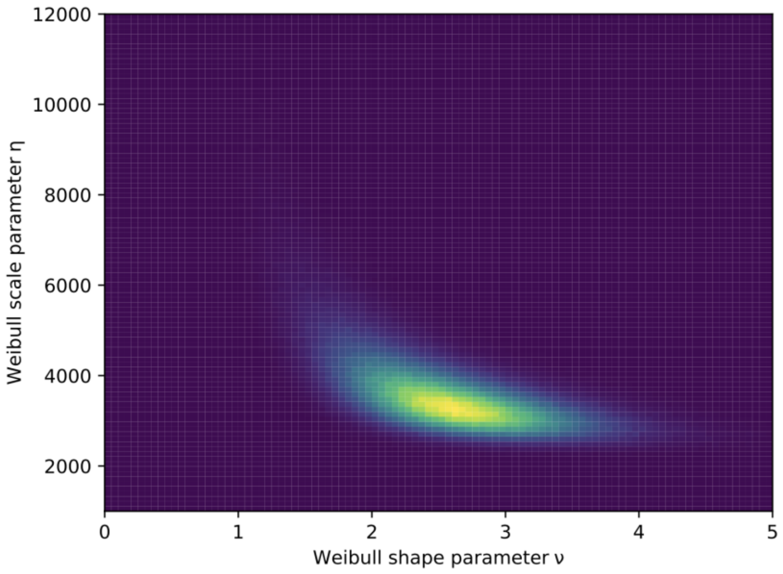
- The **MH** algorithm is used to generate a set of points in the **Weibull parameter space** (ν, η)
- Points are generated from a unbiased starting point (ν = 1 and η = 7500)
- Each new candidate point is selected via a **Gaussian random walk**
- Points that a favoured by the **Likelihood Function** are more likely to enter the dataset



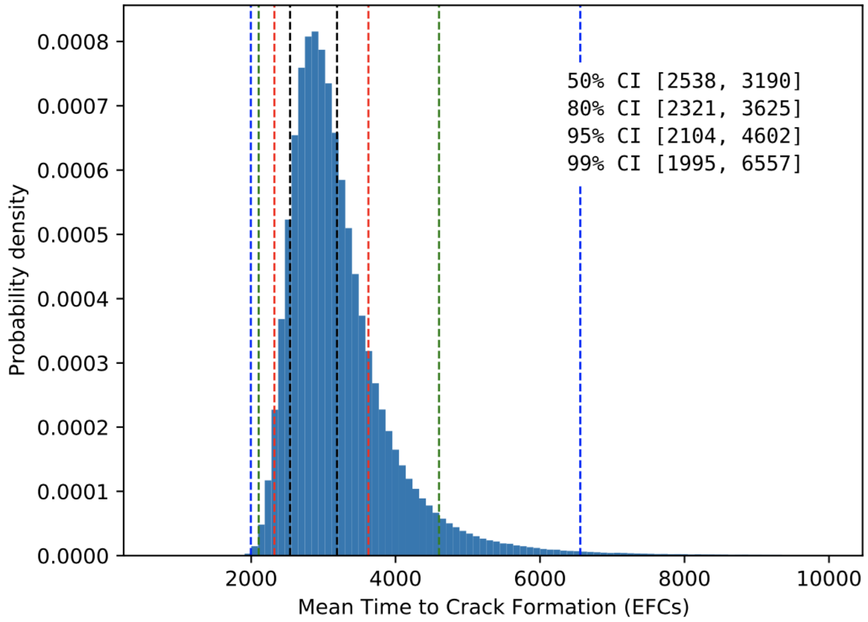
Starting point
ν = 1, η = 7500

Simulation results

The resulting simulated dataset plotted in the 2D Weibull parameter space (5M points)

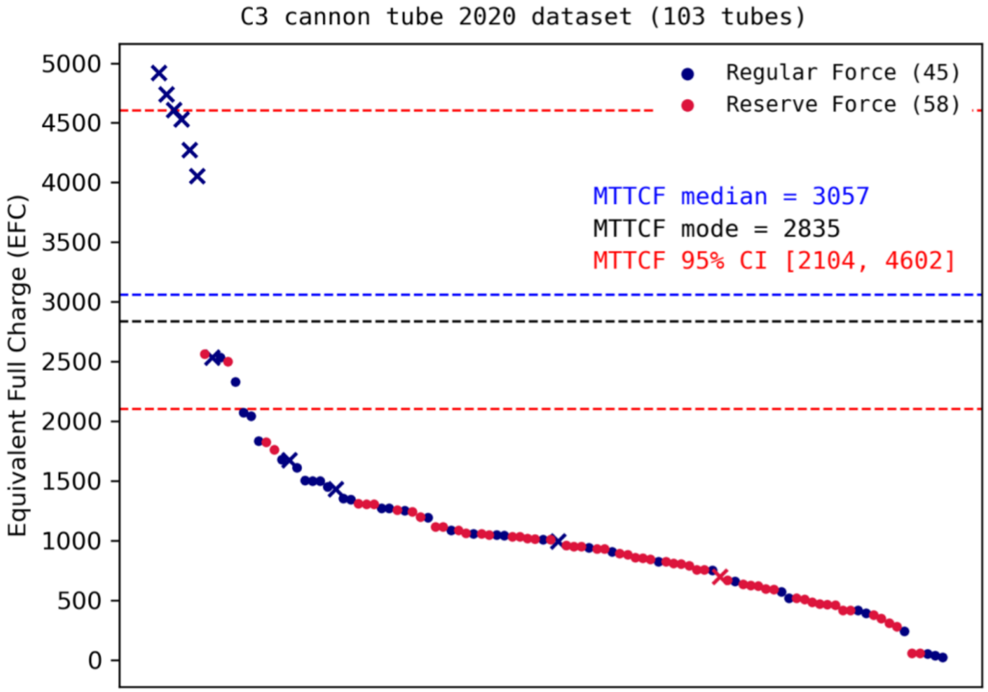


Derive the mean time to crack formation (MTTCF) distribution from the dataset



$$\text{MTTCF} = \eta \, \Gamma\left(1 + \frac{1}{\nu}\right)$$

Result - Mean time to crack formation



- The **MTTCF** is one of the main results, it represents the **expected lifetime** of the cannon tubes
- **MTTCF** analysis results:
  - Most likely value (mode) is **2835 EFCs**
  - **95% interval** ranges from **2104** to **4602** EFCs
  - **99.4%** < 7500 EFCs (rated lifetime)
- **Very few tubes** have usage within the predicted 95% interval of MTTCF
  - If more tubes enter this range and/or new cracks form it will *improve our understanding of the failure process*

Result - Remaining lifetimes

To predict the number of tubes that will **fail in the future**, the simulation can be performed ***for each individual tube*** based on it's current and projected usage

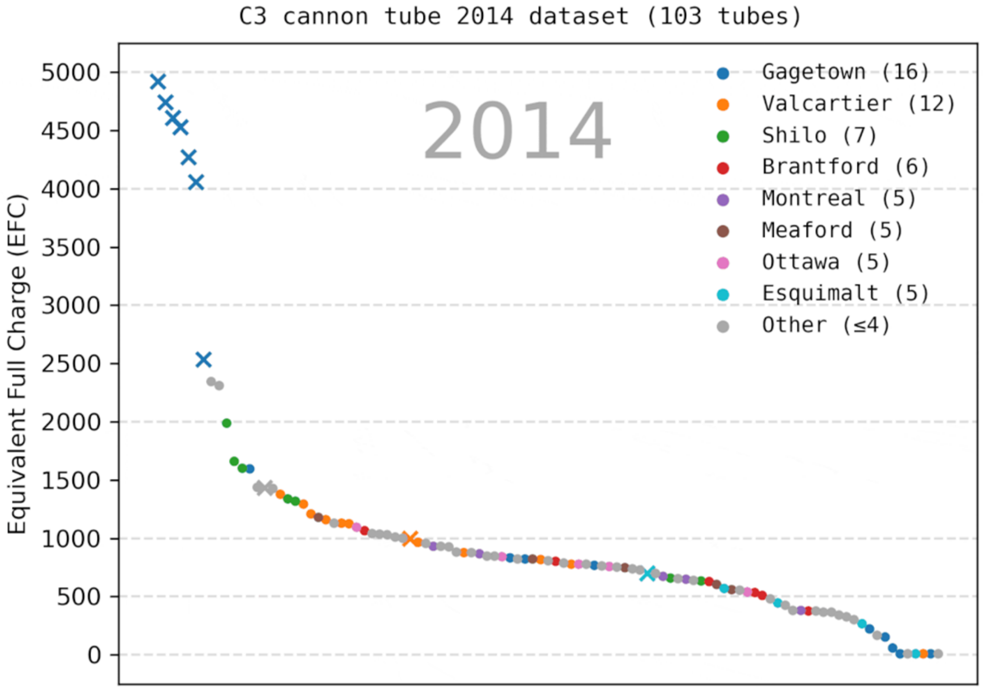
Usage / year is extrapolated based on the observed usage from **2014 -- 2020**

Result: Expect **2 tubes** to fail in **~10 years** time, and a **third** in **~15 years** time

Tube	EFCs	EFCs/year	Reg/Res	Location	Residual Life Posterior Distribution Quantiles as Crack Formation Dates: 5%	10%	20%	50%
186	2329.19	168.28	Regular	Shilo	March 2022	November 2022	February 2024	June 2028
25	2534.55	145.35	Regular	Shilo	March 2022	November 2022	March 2024	December 2028
21	1452.91	136.49	Regular	Shilo	March 2023	August 2024	February 2027	November 2033
27	1250.41	98.26	Regular	Shilo	April 2024	August 2026	May 2030	February 2040
187	661.63	84.16	Regular	Gagetown	July 2027	April 2031	August 2036	January 2049
76	1680.57	78.19	Regular	Valcartier	November 2023	December 2025	December 2029	January 2041
23	2041.90	73.23	Regular	Shilo	June 2023	March 2025	September 2028	June 2039
90	1193.79	70.45	Regular	Gagetown	August 2025	November 2028	May 2034	January 2048
89	1825.99	64.35	Reserve	Levis	January 2024	May 2026	October 2030	November 2043
37	1355.71	64.34	Regular	Valcartier	May 2025	August 2028	February 2034	October 2048
51	1498.68	61.03	Regular	Valcartier	February 2025	March 2028	August 2033	August 2048
188	414.08	59.32	Regular	Gagetown	September 2032	July 2038	July 2046	May 2064
71	1507.18	57.63	Regular	Valcartier	March 2025	July 2028	April 2034	January 2050
70	1761.76	55.66	Reserve	Shawinigan	August 2024	July 2027	September 2032	January 2048
68	949.28	51.30	Reserve	Levis	September 2028	December 2033	February 2042	October 2061
93	1304.52	50.41	Reserve	Pembroke	August 2026	December 2030	February 2038	December 2056

*the most used tubes are shown here, full table is in the report

Future projection animation



- Based on the results of the analysis, I **simulated** one possible scenario where **5 tubes fail** by the year 2040
  - Tube usage is **extrapolated** based on the observed usage between **2014** and **2020**

Summary

Quantitative Results

- **99.8%** chance that crack formation is a ***wearout*** type process
- **99.4%** chance that the ***Mean Time to Crack Formation*** **(MTTCF)** is less than **7500** EFCs
- Most likely lifetime is **2835** EFCs
- Expect **2 new crack** in the next **10** years, **3** new cracks in the next **15** years

Confirmed results of standalone simulation with RStan simulation of the same model via Hamiltonian Monte Carlo sampling.

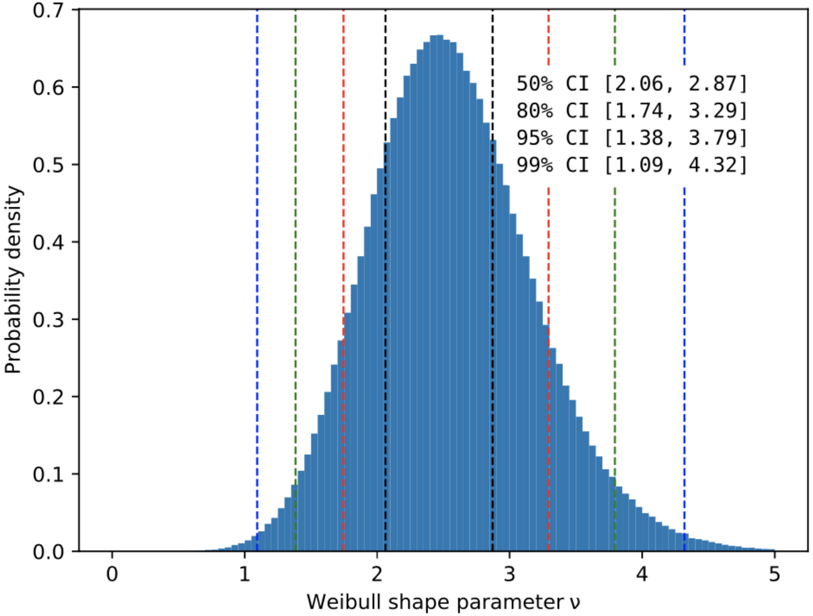
Qualitative remarks

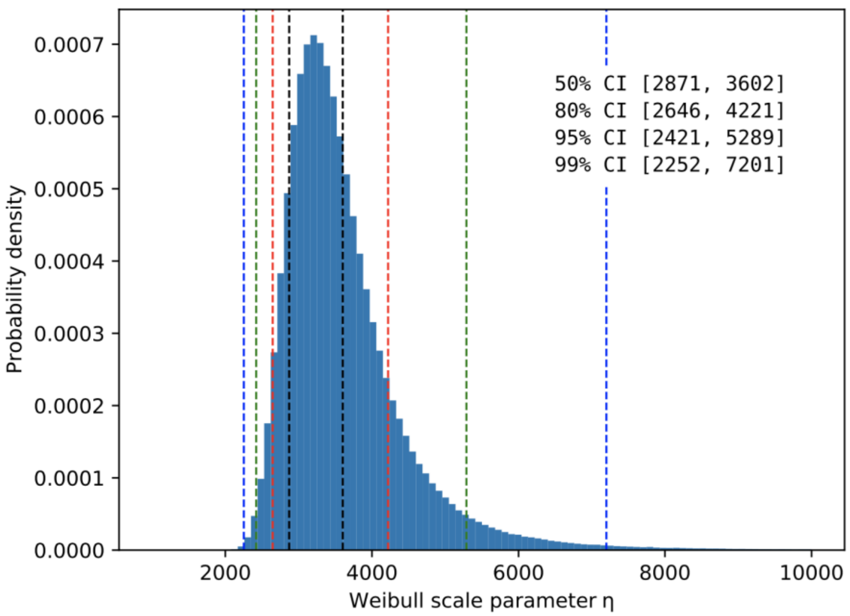
- Majority of cracked tubes (10 of 11) are with the Regular force, 8 are in **Gagetown**
- **Shilo / Brandon** tubes (10 tubes, AVCON) have high usage, *~2x the population mean*
  - A case study of the usage of these tubes located in MB would be useful to better understand the crack formation process

Questions

BACKUP

Simulated Weibull parameter distributions





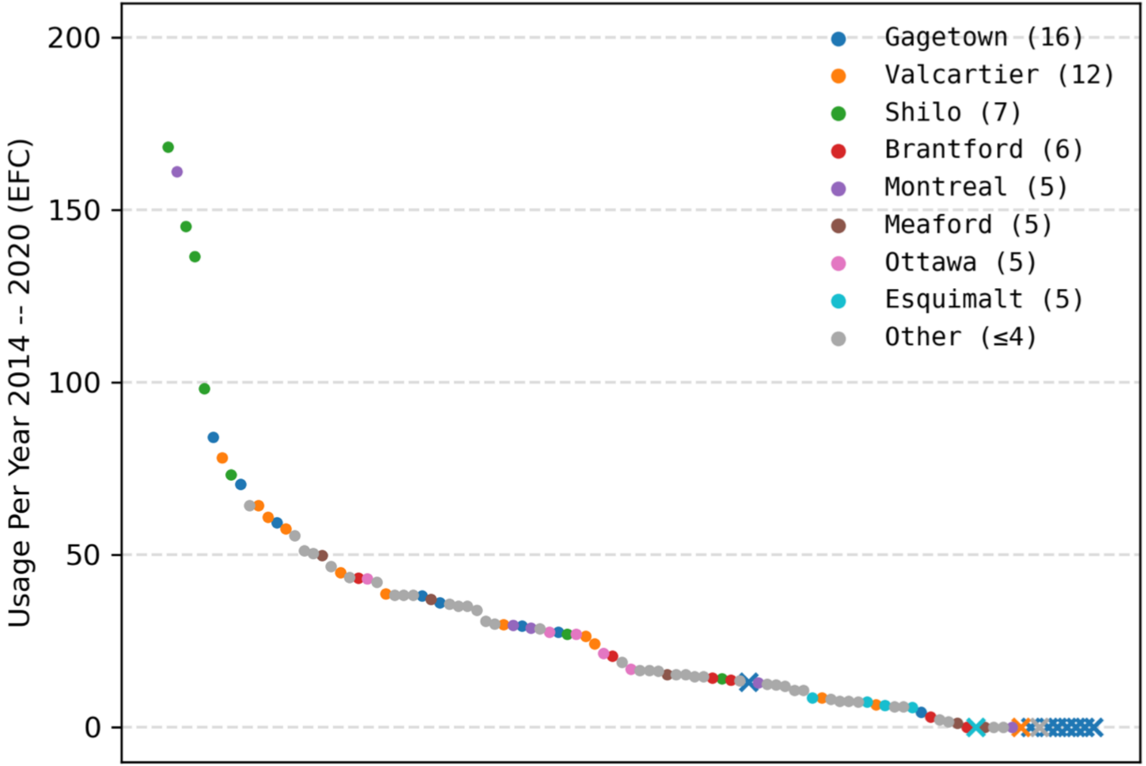
Hamiltonian Monte Carlo cross check

- MH algorithm performs adequetly for this study since the sampled Weibull parameter space is smooth and compact in two dimensions.
- Hamiltonian Monte Carlo (HMC) is a modern alternative method for sampling parameter space
  - Would outperform MH in more complex multidimensional contexts
- Used HMC as implemented via the NUTS sampler in RStan as a cross check of the results

Table 3-1: Comparison of key results derived from MCMC samples generated by the standalone implementation of the Metropolis-Hastings sampler and from the default Hamiltonian Monte Carlo sampler implemented in Rstan. All generated shape parameters from Rstan were greater than one indicating a 100% likelihood of a wear out process.

Result	Standalone MH sample	Rstan HMC NUTS sample
Likelihood of a wear out type failure process	99.8%	100%
Likelihood that the MTTCF is less than the rated lifetime of 7500 EFCs	99.4%	99.9%
MTTCF mode, median (EFC)	2835, 3057	3700, 3885
MTTCF credible intervals (EFC)	80% CI = [2321, 3625] 95% CI = [2104, 4602]	80% CI = [3242, 4395] 95% CI = [3014, 4959]

Usage per year by location



MH algorithm details

$$L(\{t_i\}|\nu,\eta) = \prod_{i \in \text{cracked}} F(t_i;\nu,\eta) \prod_{i \in \text{uncracked}} S(t_i;\nu,\eta)$$

We can perform the MCMC using the likelihood function and a process called the **Metropolis-Hastings (MH)** algorithm

MH algorithm:

- 1. Choose some **starting point** in parameter space, in our case some reasonable $\boldsymbol{\nu}$ and **η** parameters
    - Example: $\boldsymbol{\nu}$ = 1 and **η** = 7500
    - This is so it's not initially biased in terms of shape and the scale of 7500 is the rated lifetime
- 2. Generate a **new point** nearby (**$\boldsymbol{\nu}$', η'**)
    - There's different ways to generate the new points, one way is with a *random Gaussian walk*
    - This is controlled by the variance (**σ^2**) of the Gaussian distributions
    - Example: If I set $\boldsymbol{\sigma_{shape}}$ = 0.5 and $\boldsymbol{\sigma_{scale}}$ = 500
    - Then (**$\boldsymbol{\nu}$', η'**) = ($\boldsymbol{\nu}$ + Gaus(0, 0.5), **η** + Gaus(0, 500))
- 3. Compute the **likelihood ratio r = L($\{t_i\}|\nu',\eta'$) / L($\{t_i\}|\nu,\eta$)** == **L**(new point) / **L**(old point)
- 4. **Accept** or **Reject** the new point:
    - Generate a uniform random number **u** ∈ [0, 1]
    - If **u** <= **r** : *accept* the new point, it becomes the current point
    - If **u** > **r** : *reject* the new point, keep the old one
- 5. **Record** the current point
- **Repeat** steps 2 - 5 for however many samples are required

Remaining lifetime

For time t_0 in the future

$$f(t|t_0;\nu,\eta) = \frac{f(t_0+t)}{S(t_0)} = \frac{\nu}{\eta}\left(\frac{t_0+t}{\eta}\right)^{\nu-1} \exp\left(\frac{t_0^\nu - (t_0+t)^\nu}{\eta^\nu}\right)$$

Weibull PDF at time **(*t* + *t*$_0$)**

Survival function at ***t*$_0$**

- Use the sample of 1M Weibull parameter pairs to generate this distribution for each cannon tube
- For each tube, ***t*$_0$** is the *present accumulated EFC*
- We are interested in the quantiles of this distribution, particularly the **50% quantile** which will give the **median remaining lifetime** of the uncracked tubes